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# Siklos waves with torsion in 3D

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**ABSTRACT:** Starting from the Siklos waves in general relativity with a cosmological constant, interpreted as gravitational waves on the anti-de Sitter background, a new class of exact torsion waves is constructed in the framework of three-dimensional gravity with propagating torsion. In the asymptotic limit, the geometry of torsion waves takes the anti-de Sitter form. In the sector with massless torsion modes, we found a set of asymptotic conditions that leads to the conformal asymptotic symmetry.

**KEYWORDS:** Classical Theories of Gravity, Field Theories in Lower Dimensions, Conformal and W Symmetry

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## 1 Introduction

Exact gravitational waves have been an important subject of investigation in general relativity (GR) from the early 1920s; for a review, see [1–4]. Most of the activity on the subject has been focused on asymptotically flat models, the solutions of GR without a cosmological constant. From 1980s, exact gravitational waves have been studied also in GR with a cosmological constant ( $\text{GR}_\Lambda$ ) [5–7], see also [8]; for higher-dimensional extensions, see [9–13]. In particular, exact gravitational waves with an AdS asymptotic behavior attracted a lot of interest in regard to the AdS/CFT correspondence [14, 15]. Moreover, some of these solutions “may serve as exact models of the propagation of primordial gravitational waves and may be relevant for the (hypothetical) cosmological wave background” [16].

To properly understand dynamical complexities of gravity, one often relies on technically simplified three-dimensional (3D) models (for a review and an extensive list of references, see [17, 18]). In 3D, both GR and  $\text{GR}_\Lambda$  are topological theories without propagating degrees of freedom, in which nontrivial wave solutions can exist only in the presence of *matter sources* [19–21]. To avoid such a degenerate situation, one is naturally motivated to study alternative gravitational models possessing true dynamical degrees of freedom. The well-known models of this type, formulated in the context of *Riemannian geometry* of

spacetime, are Topological massive gravity and New massive gravity [22–24]. Their dynamical properties allow for the existence of gravitational waves *in vacuum*; see, for instance, Ayón-Beato et al. [25].

In the early 1960s, a new approach to gravitational dynamics was proposed, based on a modern, gauge-field-theoretic approach, known as the *Poincaré gauge theory* (PGT) (see [26, 27], for a textbook exposition of PGT, [28] for an up-to-date status of PGT, including its 3D version, and [29]) with an underlying *Riemann-Cartan (RC) geometry* of spacetime, characterized by both the *curvature* and the *torsion*. In a topological version of the three-dimensional PGT, gravitational waves with torsion were constructed in the presence of matter sources by Obukhov [30]. However, genuine gravitational waves are those that can propagate in spacetime regions without matter. Further investigations of the PGT, with a Lagrangian that is at most quadratic in the field strengths (quadratic PGT), revealed a rich dynamical structure, expressed, in particular, by the existence of propagating torsion modes [31]. In a recent paper [32],<sup>1</sup> we used quadratic PGT to construct exact torsion waves in vacuum as a generalization of the plane-fronted waves from GR.

In the present paper, we continue the investigation of genuine gravitational waves with torsion in 3D, by focusing on the anti-de Sitter (AdS) background. We found a new class of exact torsion waves in vacuum, representing a PGT extension of the Siklos waves in  $\text{GR}_\Lambda$  [33], see also [8, 16]. In the linear approximation, this class is associated to spin-2 torsion excitations around the AdS background. In the sector of massless torsion modes, we found a set of asymptotic conditions that leads to a conformal asymptotic symmetry, characterized by two independent Virasoro algebras with central charges. On the other hand, massive torsion waves show kind of an oscillatory behavior in the asymptotic region.

The paper is organized as follows. In section 2, we give an overview of the Siklos waves in the three-dimensional  $\text{GR}_\Lambda$ . In section 3, we construct a new wave solution in PGT, taking the metric to be of the Siklos form, whereas the torsion piece of the connection is assumed to possess only the tensorial irreducible component. The solutions of the field equations are found and classified according to the values of the mass parameter  $\mu^2$ , associated to the spin-2 torsion modes. For  $\mu^2 \geq 0$  (no tachyons), the asymptotic limit of the Siklos waves with torsion is shown to be represented by Riemannian AdS spacetimes. In section 4, we study the form of the AdS asymptotic conditions for  $\mu^2 \geq 0$ . It turns out that a well-defined asymptotic structure exists only in the massless sector. The corresponding central charges of the asymptotic symmetry are found in section 5, and section 6 is devoted to concluding remarks. Finally, two appendices contain some technical details.

Here are our conventions: the Latin indices  $(i, j, k, \dots)$  refer to the local Lorentz (co)frame and run over  $(+, -, 2)$ ,  $b^i$  is the triad field (coframe 1-form),  $h_i$  is the dual basis (frame), totally antisymmetric tensor  $\varepsilon^{ijk}$  is normalized to  $\varepsilon^{+-2} = 1$ ; the Greek indices  $(\mu, \nu, \rho, \dots)$  refer to the coordinate frame; the Lie dual of an antisymmetric form  $X^{jk}$  is  $X_i := -\varepsilon_{ijk} X^{jk}/2$ , the Hodge dual of a form  $\alpha$  is  ${}^*\alpha$ , and the exterior product of forms is implicit.

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<sup>1</sup>Here, the reader can find references of earlier studies of exact gravitational waves with torsion in 4D.

## 2 Siklos waves

In 1980s, Siklos [33] found a special class of exact gravitational waves propagating on the AdS background, the physical interpretation of which was investigated in detail by Podolský [16]. In the Poincaré coordinates  $x^\mu = (u, v, y)$ , the Siklos metric in 3D has the form

$$ds^2 = \frac{\ell^2}{y^2} [2du(Hdu + dv) - dy^2] , \quad (2.1)$$

with  $H = H(u, y)$ , which is equivalent to a subclass of the Kundt metric [8, 16]. The wave fronts are labeled by  $u = \text{const.}$ ,  $v$  is an affine parameter along the corresponding rays generated by the Killing vector field  $\partial_v$  that is null but not covariantly constant, and for  $H = 0$  the metric reduces to the AdS background (see appendix A). We choose the triad field  $b^i$  (1-form) to be

$$b^+ := \frac{\ell}{y} du, \quad b^- := \frac{\ell}{y} (Hdu + dv), \quad b^2 = \frac{\ell}{y} dy, \quad (2.2)$$

so that the line element is given by  $ds^2 = \eta_{ij} b^i b^j$ , with the half-null Lorentz metric

$$\eta_{ij} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} .$$

The dual frame basis  $h_i$ , defined by  $h_i \lrcorner b^j = \delta_i^j$ , is given by

$$h_+ = \frac{y}{\ell} (\partial_u - H \partial_v), \quad h_- = \frac{y}{\ell} \partial_v, \quad h_2 = \frac{y}{\ell} \partial_y .$$

The related Riemannian connection  $\omega^{ij}$  (1-form) can be written in a compact form as

$$\omega^{ij} = \bar{\omega}^{ij} - \frac{1}{\ell} \varepsilon^{ij}_m k^m (yH') k_n b^n . \quad (2.3a)$$

Here, prime denotes a derivative with respect to  $y$ , the first term  $\bar{\omega}^{ij}$  describes the background AdS geometry,

$$\bar{\omega}^{+-} = 0, \quad \bar{\omega}^{+2} = \frac{1}{\ell} b^+, \quad \bar{\omega}^{-2} = \frac{1}{\ell} b^-, \quad (2.3b)$$

and the second one is the radiation piece, characterized by the null vector  $k^m := (0, 1, 0)$ , with  $k_m = (1, 0, 0)$ .

Next, we calculate the Riemannian curvature,

$$R^{ij} = \frac{1}{\ell^2} b^i b^j - \frac{1}{\ell^2} \varepsilon^{ij}_m k^m (y^2 H'' - yH') k_n b^n, \quad (2.4a)$$

whereupon the Ricci curvature  $(Ric)^i = -h_j \lrcorner R^{ij}$  and the scalar curvature  $R = h_i \lrcorner (Ric)^i$  are found to be

$$\begin{aligned} (Ric)^i &= \frac{2}{\ell^2} b^i + \frac{1}{\ell^2} k^i (y^2 H'' - yH') k_n b^n, \\ R &= \frac{6}{\ell^2}. \end{aligned} \quad (2.4b)$$

When the Siklos metric satisfies the vacuum field equation of  $\text{GR}_\Lambda$  with  $\Lambda \sim -1/\ell^2$ , the metric function  $H$  takes a simple form:

$$y^2 H'' - yH' = 0 \quad \Rightarrow \quad H = D_1(u) + D_2(u)y^2. \quad (2.5)$$

However, this solution is *trivial*. Indeed, since the radiation piece of the curvature vanishes on shell, we have  $R^{ij} = b^i b^j / \ell^2$ , and the geometry of spacetime is fixed, it has the AdS form. Nontrivial AdS waves can exist in  $\text{GR}_\Lambda$  only in the presence of matter [19–21], but to have *vacuum* AdS waves, one has to change the gravitational dynamics. As we shall see, transition to quadratic PGT allows the existence of genuine AdS waves with torsion.

### 3 Siklos waves with torsion

Basic gravitational variables of PGT are the triad field  $b^i$  and the Lorentz connection  $\omega^{ij}$  (1-forms), and the related field strengths are the torsion  $T^i = db^i + \omega^i_m b^m$  and the curvature  $R^{ij} = d\omega^{ij} + \omega^i_m \omega^{mj}$  (2-forms). Relying on PGT, we now introduce a geometric extension of the Siklos waves (2.1) to genuine Siklos waves with torsion.

#### 3.1 Ansatz

In order to *preserve the radiation nature* of the Siklos metric, we assume that the form of the triad field in PGT remains the same as in eq. (2.2). Essentially the same idea can be applied also to the connection [32]: starting from the Riemannian connection (2.3), we assume that the new, RC connection is given by

$$\omega^{ij} = \bar{\omega}^{ij} - \frac{1}{\ell} \varepsilon^{ij}_m k^m (yG) k_n b^n, \quad (3.1a)$$

where

$$G := H' + K, \quad K = K(u, y). \quad (3.1b)$$

Geometrically, the new function  $K$  in the connection is related to the torsion:

$$T^i := \nabla b^i = -\frac{yK}{\ell} k^i k^{n*} b_n. \quad (3.2)$$

For  $K = 0$ , the torsion vanishes, and the connection becomes equivalent to  $\bar{\omega}^{ij}$ . The only nonvanishing irreducible component of  $T^i$  is its tensorial piece  $^{(1)}T^i$  [32], so that

$$^{(1)}T^i = T^i.$$

Using the above ansatz for the connection, one can calculate the RC curvatures:

$$\begin{aligned} R^{ij} &= \frac{1}{\ell^2} b^i b^j - \frac{1}{\ell^2} \varepsilon^{ij}_m k^m (y^2 G' - yH') k^{n*} b_n, \\ (Ric)^i &= \frac{2}{\ell^2} b^i + \frac{1}{\ell^2} k^i (y^2 G' - yH') k_n b^n, \\ R &= \frac{6}{\ell^2}. \end{aligned} \quad (3.3)$$

The quadratic curvature invariant takes the form

$$R^{ij\star}R_{ij} = \frac{6}{\ell^4}\star 1.$$

The only nonvanishing irreducible components of  $R^{ij}$  are:

$${}^{(6)}R^{ij} = \frac{1}{6}Rb^ib^j, \quad {}^{(4)}R^{ij} = R^{ij} - {}^{(6)}R^{ij}.$$

For more details on the irreducible decomposition of the field strengths, see ref. [32].

In what follows, the specific forms of both the metric function  $H$  and the torsion function  $K$  will be determined by the PGT field equations.

### 3.2 Lagrangian dynamics of PGT

The PGT dynamics is described by a Lagrangian 3-form  $L_G = L_G(b^i, T^i, R^{ij})$ , which is assumed to be at most quadratic in the field strengths (quadratic PGT) and parity invariant. In conformity with our ansatz, the Lagrangian is chosen to have the form

$$\begin{aligned} L_G = & -a_0\varepsilon_{ijk}b^iR^{jk} - \frac{1}{3}\Lambda_0\varepsilon_{ijk}b^ib^jb^k \\ & + T^i\star\left(a_1{}^{(1)}T_i\right) + \frac{1}{2}R^{ij}\star\left(b_4{}^{(4)}R^{ij} + b_6{}^{(6)}R^{ij}\right). \end{aligned} \quad (3.4)$$

Indeed, the only nonvanishing irreducible components of the field strengths appearing in  $L_G$  are  ${}^{(1)}T^i$ ,  ${}^{(4)}R^{ij}$  and  ${}^{(6)}R^{ij}$ , and  $a_1, b_4, b_6$  are the corresponding coupling constants. Then, the PGT field equations in vacuum are found to be (see section III.A of ref. [32]):

$$\begin{aligned} (1ST): \quad & (a_0\ell^2 - b_4 - b_6)(yH'' - H') + (a_0\ell^2 - a_1\ell^2 - b_4 - b_6)yK' = 0, \\ & 2a_0\ell^2 + b_6 + 2\ell^4\Lambda_0 = 0, \\ (2ND): \quad & b_4[y^2(H''' + K'') + yK'] - (a_0\ell^2 - a_1\ell^2 - b_6)K = 0. \end{aligned} \quad (3.5)$$

These equations are checked using the Excalc package of the computer algebra system Reduce. Using the expression for  $(1ST)'$ , one finds that  $(2ND)$  can be rewritten as

$$y^2K'' + yK' + \ell^2\mu^2K = 0, \quad \mu^2 = \frac{(a_1 - a_0 - b_6\lambda)(a_0 + b_4\lambda + b_6\lambda)}{b_4a_1},$$

where  $\lambda := -1/\ell^2$ . Finally, after introducing the notation

$$\hat{y} = \frac{y}{\ell}, \quad m^2 = \ell^2\mu^2,$$

the two field equations take a more compact form:

$$\begin{aligned} (1ST) \quad & \hat{y}H'' - H' = \ell C\hat{y}K', \quad C := \frac{a_1}{a_0 + b_4\lambda + b_6\lambda} - 1, \\ (2ND) \quad & \hat{y}^2K'' + \hat{y}K' + m^2K = 0, \end{aligned} \quad (3.6)$$

where prime now denotes differentiation with respect to  $\hat{y}$ . As one can see, it is the presence of torsion ( $K \neq 0$ ) that makes the metric of the AdS wave nontrivial ( $\hat{y}H'' - H' \neq 0$ ). Equations (3.6) define a new class of Siklos waves — the Siklos waves with torsion.

### 3.3 Solutions

The coefficient  $m^2$  in  $(2ND)$  is the (dimensionless) mass parameter associated to the spin-2 excitation of the torsion field around the AdS background, see [31, 32]. The absence of tachyons requires  $m^2 \geq 0$ . In this subsection, we construct the exact Sikos waves with torsion, and classify them according to the values of  $m^2$ .

**(1)  $m^2 > 0$ .** The Euler (or Euler-Fuchs, Euler-Cauchy) differential equation  $(2ND)$  is solved by the ansatz  $K = \hat{y}^\alpha$ , which yields  $\alpha^2 + m^2 = 0$ . For  $m^2 > 0$ , we have  $\alpha = \pm im$ , so that  $K = \hat{y}^{\pm im} = e^{\pm im \ln \hat{y}}$ , or equivalently,

$$K = A(u) \cos(m \ln \hat{y}) + B(u) \sin(m \ln \hat{y}). \quad (3.7a)$$

By substituting this result into  $(1ST)$ , one finds the related solution for  $H$ :

$$H = D_1 + D_2 \hat{y}^2 + \frac{\ell C m}{1 + m^2} \hat{y} [A(u) \sin(m \ln \hat{y}) - B(u) \cos(m \ln \hat{y})]. \quad (3.7b)$$

The first two terms, which represent a solution of the homogeneous equation  $\hat{y} H'' - H' = 0$ , can be geometrically disregarded, as they do not influence the values of the field strengths.

In the asymptotic limit  $\hat{y} \rightarrow 0$ , the torsion and the radiation piece of the curvature,  ${}^{(4)}R_{ij}$ , vanish, as follows from the relations

$$\begin{aligned} \lim_{\hat{y} \rightarrow 0} \hat{y} K &= 0, \\ \lim_{\hat{y} \rightarrow 0} [\hat{y}^2 (H'' + K') - \hat{y} H'] &= \lim_{\hat{y} \rightarrow 0} [\hat{y}^2 K' + \ell C \hat{y}^2 K'] = 0. \end{aligned} \quad (3.8)$$

Thus, the asymptotic geometry of our solution is given by the Riemannian AdS spacetime.

**(2)  $m^2 = 0$ .** In order to have a smooth Minkowskian limit for  $\ell^2 \rightarrow \infty$ , the condition  $m^2 = 0$  is realized by demanding [31]

$$a_1 - a_0 + b_6/\ell^2 = 0. \quad (3.9)$$

As a consequence, the solution for the massless torsion wave is given by

$$\begin{aligned} K &= C_1 + C_2 \ln \hat{y}, \\ H &= D_1 + D_2 \hat{y}^2 - \ell C C_2 \hat{y}. \end{aligned} \quad (3.10)$$

As before, one can choose  $D_1 = D_2 = 0$  without loss of generality, so that the asymptotic limit of the solution is again given by the Riemannian AdS spacetime.

**(3)  $m^2 < 0$ .** Although the spin-2 torsion modes are now tachyons, we present the related exact wave solution, for the sake of completeness:

$$\begin{aligned} K &= A \hat{y}^m + B \hat{y}^{-m}, \\ H &= \frac{\ell C m}{m^2 - 1} (A \hat{y}^{1+m} - B \hat{y}^{1-m}). \end{aligned} \quad (3.11)$$

The asymptotic behavior depends on the value of  $m$ .

## 4 Asymptotic conditions

In our study of the asymptotic conditions, we assume that the topology of the spacetime manifold  $M$  is  $R \times \Sigma$ , where  $R$  is interpreted as time, and  $\Sigma$  is a spatial section of spacetime, whose boundary  $\partial\Sigma$  is topologically a circle. The asymptotic analysis is simplified by introducing a new set of local coordinates  $(t, \varphi)$ , given by  $u = (t + \ell\varphi)/\sqrt{2}$ ,  $v = (t - \ell\varphi)/\sqrt{2}$ , such that the boundary  $\partial\Sigma$  at  $y = 0$  is parametrized by the angular coordinate  $\varphi$ .

As we have seen in the previous section, in the asymptotic limit  $y \rightarrow 0$ , the geometry of our torsion wave is described by the Riemannian AdS spacetime. This property motivates us to examine asymptotic conditions based on the following requirements:

- (a) asymptotic configurations include the torsion wave geometry;
- (b) they are invariant under the action of the AdS group  $SO(2, 2)$ ;
- (c) asymptotic symmetries have well defined canonical generators.

Specific aspects of these criteria depend on the value of the mass parameter  $\mu^2$ .

### 4.1 Massive torsion waves

For  $\mu^2 > 0$ , the characteristic functions  $H$  and  $K$  can be represented in the form

$$H = yW_0, \quad K = W_0, \quad (4.1a)$$

where  $W_0$  is a *generic* wave “oscillatory” function,

$$W_0 := C_1(u) \cos(m \ln y/\ell) + C_2(u) \sin(m \ln y/\ell). \quad (4.1b)$$

In spite of this oscillatory behavior, both the torsion and the wave piece of the curvature tend to zero when  $y \rightarrow 0$ .

In the matrix notation, the components of the Siklos metric (2.1) read

$$g_{\mu\nu} = \frac{\ell^2}{y^2} \begin{pmatrix} 2H & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Asymptotically, for  $y \rightarrow 0$ , we have  $g_{uu} \sim W_0/y$ , so that, to leading order in  $1/y$ ,  $g_{\mu\nu}$  reduces to the AdS metric  $\bar{g}_{\mu\nu}$ . In the asymptotic analysis, we use  $\mathcal{O}(y^n W_0)$  to denote a term that is *at most* proportional to  $y^n W_0$  when  $y \rightarrow 0$ . Thus, the Siklos metric is of the type

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + G_{\mu\nu}, \quad G_{\mu\nu} := \begin{pmatrix} \mathcal{O}(W_0/y) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Looking at the action of the AdS Killing vectors (appendix A) on  $g_{\mu\nu}$ , one finds that the general requirements (a) and (b) are fulfilled by the following asymptotic configurations:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + G_{\mu\nu}, \quad G_{\mu\nu} := \begin{pmatrix} \mathcal{O}_{-1} & \mathcal{O}_0 & \mathcal{O}_0 \\ \mathcal{O}_0 & \mathcal{O}_0 & \mathcal{O}_0 \\ \mathcal{O}_0 & \mathcal{O}_0 & \mathcal{O}_0 \end{pmatrix}, \quad (4.2)$$



where  $\mathcal{O}_n := \mathcal{O}(y^n W_0)$ . The asymptotic form (4.2), but with  $\mathcal{O}_n = \mathcal{O}(y^n)$ , was studied earlier by Afshar et al. [34, 35], in the context of Conformal Chern-Simons gravity.

The asymptotic conditions (4.2) are preserved by the local translations of the form

$$\begin{aligned}\xi^u &= \varepsilon^u(u) + \frac{y^2}{4} \partial_v^2 \varepsilon^v(v) + \mathcal{O}_3, \\ \xi^v &= \varepsilon^v(v) + \frac{y^2}{4} \partial_u^2 \varepsilon^u(u) + \mathcal{O}_3, \\ \xi^2 &= \frac{y}{2} (\partial_u \varepsilon^u + \partial_v \varepsilon^v) + \mathcal{O}_3.\end{aligned}\tag{4.3}$$

These parameters are essentially of the Brown-Henneaux type [35, 36].

In the next step, one could try to extend these considerations to the variables  $b^i$  and  $\omega^{ij}$ . However, a problem arises when we return to our general requirement (c). Namely, although the field strengths  $T^i$  and  $R^{ij}$  have an AdS asymptotic limit, the asymptotic behavior of  $b^i$  and  $\omega^{ij}$  is determined by the function  $W_0$ , which oscillates when  $y \rightarrow 0$ . Thus, the basic dynamical variables have *no asymptotic limit*, and one is not able to define surface terms of the canonical generators. Thus, one cannot formulate a boundary theory, and in particular, the AdS/CFT correspondence is not well defined.

## 4.2 Massless torsion waves

In the sector with massless torsion modes, the form of our wave solution is displayed in eq. (3.10). As we noted before, the geometrically irrelevant term  $D_1 + D_2 y^2$  in  $H$  can be removed by choosing  $D_1 = D_2 = 0$ , whereupon the characteristic functions  $H$  and  $K$  are of the generic form

$$H = C_0(y/\ell), \quad K = C_1 + C_2 \ln(y/\ell).\tag{4.4}$$

The asymptotic geometry of the solution is described by the AdS spacetime. In this section, we discuss the asymptotic structure of the massless torsion wave (4.4).

Quite generally, the wave triad (2.2) can be written in the form  $b^i{}_\mu = \bar{b}^i{}_\mu + B^i{}_\mu$ , where  $\bar{b}^i$  is the AdS triad, and the only nonvanishing component of  $B^i{}_\mu$  is  $B^-{}_u = \ell H/y = C_0$ . Then, in accordance with the general requirements (a) and (b), we choose the following asymptotic form of the triad field:

$$b^i{}_\mu = \bar{b}^i{}_\mu + B^i{}_\mu, \quad B^i{}_\mu := \begin{pmatrix} \mathcal{O}_1 & \mathcal{O}_1 & \mathcal{O}_1 \\ \mathcal{O}_0 & \mathcal{O}_1 & \mathcal{O}_1 \\ \mathcal{O}_1 & \mathcal{O}_1 & \mathcal{O}_1 \end{pmatrix},\tag{4.5}$$

where  $\mathcal{O}_n := \mathcal{O}(y^n)$ . These conditions impose the following restriction on the local Poincaré parameters  $(\xi^\rho, \varepsilon^{ij})$ :

$$\delta_0 b^i{}_\mu := \varepsilon^{ijk} \theta_j b_{k\mu} - (\partial_\mu \xi^\rho) b^i{}_\rho - \xi^\rho \partial_\rho b^i{}_\mu = B^i{}_\mu,$$

where  $\theta^i$  is the Lie dual of  $\varepsilon_{mn}$ . As a consequence, the asymptotic parameters of local translations take the form displayed in eq. (4.3), whereas the asymptotic parameters of

Lorentz rotations are found to be

$$\begin{aligned}\theta^+ &= \frac{y}{2} \partial_v^2 \varepsilon^v + \mathcal{O}_2, \\ \theta^- &= -\frac{y}{2} \partial_u^2 \varepsilon^u + \mathcal{O}_2, \\ \theta^2 &= \frac{1}{2} (\partial_v \varepsilon^v - \partial_u \varepsilon^u) + \mathcal{O}_2.\end{aligned}\tag{4.6}$$

Next, we wish to examine whether the asymptotic behavior of the RC connection (3.1) can be made compatible with the already found form of the asymptotic Poincaré parameters. First, we introduce the Lie-dual connection  $\omega^i$ :

$$\omega^+ = \frac{1}{\ell} b^+, \quad \omega^- = -\frac{1}{\ell} b^- + \frac{y}{\ell} G b^+, \quad \omega^2 = 0.\tag{4.7}$$

The form of  $K$  implies that the asymptotic conditions on the connection should contain log terms. By combining the expression (4.7) for  $\omega^i_\mu$  with the asymptotic formulas for  $b^\pm$  and  $G = H' + K$ , we find it suitable to assume

$$\omega^i_\mu = \bar{\omega}^i_\mu + \Omega^i_\mu, \quad \Omega^i_\mu := \frac{1}{\ell} \begin{pmatrix} \mathcal{O}_1 & \mathcal{O}_1 & \mathcal{O}_1 \\ \mathcal{O}(\ln y/\ell) & \mathcal{O}_1 & \mathcal{O}(y \ln y/\ell) \\ \mathcal{O}(y \ln y/\ell) & \mathcal{O}_1 & \mathcal{O}_1 \end{pmatrix}.\tag{4.8}$$

As it turns out, the asymptotic invariance of  $\omega^i_\mu$ ,

$$\delta_0 \omega^i_\mu := -\partial_\mu \theta^i - \varepsilon^{ijk} \omega_{j\mu} \theta_k - \partial_\mu \xi^\rho \omega^i_\rho - \xi^\rho \partial_\rho \omega^i_\mu = \Omega^i_\mu,$$

does not impose any new restriction of the asymptotic Poincaré parameters (4.3) and (4.6).

In order to clarify the interpretation of our asymptotic conditions, we wish to find the commutator algebra of the asymptotic Poincaré transformations. To do that, we note that the composition law of the asymptotic transformations, to lowest order in  $y$ , reads

$$(\varepsilon^u)''' = (\varepsilon^u)' \partial_u (\varepsilon^u)'' - (\varepsilon^u)'' \partial_u (\varepsilon^u)',\tag{4.9}$$

and similarly for  $\varepsilon^v$ . Then, introducing the notation

$$\ell_n^+ := -\frac{1}{\sqrt{2}} \delta_0 \left( \varepsilon^u = \ell e^{inu\sqrt{2}/\ell}, \varepsilon^v = 0 \right), \quad \ell_n^- := -\frac{1}{\sqrt{2}} \delta_0 \left( \varepsilon^u = 0, \varepsilon^v = \ell e^{inv\sqrt{2}/\ell} \right),$$

the commutator algebra of the asymptotic symmetry takes the form of two independent Virasoro algebras:

$$i [\ell_m^\pm, \ell_n^\pm] = (m-n) \ell_{m+n}^\pm.\tag{4.10}$$

The related central charges are discussed in the next section.

## 5 Canonical form of the asymptotic symmetry

In this section, we use the canonical approach to analyze the asymptotic symmetry in the massless sector, including the values of the central charges.

To simplify the analysis, we follow Nester [37] in applying the *first-order formulation* to the quadratic PGT. In this formalism, the Lagrangian (3.4) is written in the form

$$L_G = T^i \tau_i + \frac{1}{2} R^{ij} \rho_{ij} - V(b^i, \tau_i, \rho_{ij}) - \frac{1}{3} \Lambda \varepsilon_{ijk} b^i b^j b^k. \quad (5.1)$$

Here,  $\tau_i$  and  $\rho_{ij}$  are new, *independent* variables, and  $V$  is a function quadratic in  $\tau_i$  and  $\rho_{ij}$ , chosen so that, on shell, we have  $\tau_i = H_i$  and  $\rho_{ij} = H_{ij}$ , where  $H_i = \partial L_G / \partial T^i$  and  $H_{ij} = \partial L_G / \partial R^{ij}$  are the covariant field momenta associated to the original Lagrangian (3.4). Explicit form of  $V$  is described in ref. [31], and it ensures the first-order formulation (5.1) to be *equivalent* to (3.4). Thus, the variation of  $L_G$  with respect to  $\tau_i$  and  $\rho_{ij}$  yields

$$\begin{aligned} \tau_i &= 2a_1 {}^\star T_i, \\ \rho_{ij} &= -2 \left( a_0 - \frac{1}{6} b_6 R \right) \varepsilon_{ijk} b^k + 2b_4 {}^\star ({}^{(4)} R)_{ij}, \end{aligned} \quad (5.2)$$

in accordance with the forms of  $H_i$  and  $H_{ij}$  defined by the Lagrangian (3.4).

Asymptotic symmetries are best described in the canonical formalism. In the first order formulation of PGT, the canonical gauge generator is a functional  $G[\varphi, \pi]$  on the phase space, the form of which is defined in eqs. (5.7) of ref. [31]. The canonical generator acts on the phase-space variables  $(\varphi, \pi)$  via the Poisson (or Dirac) bracket operation, defined in terms of the functional derivatives. A functional  $F[\varphi, \pi] = \int d^2x f(\varphi, \partial_\alpha \varphi, \pi, \partial_\alpha \pi)$  is differentiable (or regular) if its variation has the form  $\delta F = \int d^2x [A(x) \delta \varphi + B(x) \delta \pi]$ . In order to ensure this property for our generator  $G$ , we have to improve its form by adding an appropriate surface term  $\Gamma$  [38]. The improved canonical generator  $\tilde{G} := G + \Gamma$  has been calculated in appendix B; it is both finite and differentiable (well-defined).

The Poisson bracket (PB) algebra of the improved generators could be found by a direct calculation, but we rather rely on another, more instructive method. Introducing a convenient notation,  $\tilde{G}' = \tilde{G}[\varepsilon^{u'}, \varepsilon^{v'}]$  and similarly for  $\tilde{G}''$  and  $\tilde{G}'''$ , we use the main theorem of ref. [39], which states that the PB of two well-defined generators must also be a well-defined generator, to conclude that the PB algebra has the form

$$\{\tilde{G}'', \tilde{G}'\} = \tilde{G}''' + C'''. \quad (5.3a)$$

Here, the parameters of  $\tilde{G}'''$  are defined by the composition law (4.9), and  $C'''$  is the central charge of the algebra. A simple reformulation of this formula, given by

$$\{\tilde{G}'', \tilde{G}'\} = \delta'_0 \tilde{G}'' \approx \delta'_0 \Gamma'', \quad (5.3b)$$

represents a powerful tool for calculating the central charge. Indeed, the previous two equations imply

$$\delta'_0 \Gamma'' \approx \Gamma''' + C'''. \quad (5.3c)$$

Now, since  $C'''$  does not depend on the basic dynamical variables and  $\Gamma'''$  vanishes on the AdS background (see appendix B), the evaluation of  $\delta'_0 \Gamma''$  on the AdS background yields the final expression for  $C'''$ :

$$\overline{\delta'_0 \Gamma''} = C'''. \quad (5.4)$$

An explicit calculation based on the results of appendix B yields

$$\frac{\sqrt{2}}{\ell} C''' = - \left( a_0 - \frac{b_6}{\ell^2} \right) \int_0^{2\pi} d\varphi \left( \varepsilon^{u''} \partial_u^3 \varepsilon^{u'} + \varepsilon^{v''} \partial_v^3 \varepsilon^{v'} \right). \quad (5.5)$$

This result, combined with eq. (5.3a), completes the derivation of the canonical PB algebra.

A more familiar form of this algebra is obtained by introducing the Fourier modes of the improved generator:

$$L_n^+ := -\frac{1}{\sqrt{2}} \tilde{G} \left( \varepsilon^u = \ell e^{inu\sqrt{2}/l}, \varepsilon^v = 0 \right), \quad L_n^- := -\frac{1}{\sqrt{2}} \tilde{G} \left( \varepsilon^v = \ell e^{inv\sqrt{2}/l}, \varepsilon^u = 0 \right).$$

Then, the canonical algebra (5.3a) takes the form of two independent Virasoro algebras with central charges,

$$i \{ L_m^\pm, L_n^\pm \} = (m - n) L_{m+n}^\pm + \frac{c^\pm}{12} m^3 \delta_{m+n}, \quad (5.6)$$

where the central charges are equal to each other:

$$c^\pm = \left( 1 - \frac{b_6}{a_0 \ell^2} \right) c_0. \quad (5.7)$$

Note that the coupling constant  $b_6$  modifies the  $\text{GR}_\Lambda$  central charge  $c_0 := 3\ell/2G$ , and for  $b_6 < a_0 \ell^2$ , the central charge  $c^\pm$  is positive.

## 6 Concluding remarks

In this paper, we found a new class of exact vacuum solutions of the three-dimensional PGT, the class of Siklos waves with torsion. Asymptotic geometry of these solutions is described by the Riemannian AdS spacetime. In the sector of massless torsion modes, we found a set of asymptotic conditions for which the asymptotic symmetry is described by two independent Virasoro algebras with equal central charges  $c^\pm$ , the values of which differ from the  $\text{GR}_\Lambda$  result.

Further studies of the massless sector might help us to clarify the role of torsion in the AdS/CFT correspondence.

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## A AdS and Siklos spacetimes in 3D

In this appendix, we review basic aspects of the three-dimensional AdS and Siklos spacetimes; see for instance [4, 40, 41] and [7, 8, 16], respectively.

The AdS space in 3D, with topology  $S^1 \times R^2$ , can be defined in terms of the hypersurface

$$H_3 : \quad \bar{u}^2 - \bar{x}^2 - \bar{y}^2 + \bar{v}^2 = \ell^2,$$

embedded in a 4-dimensional Minkowski space with metric  $\eta_{ab} = (1, -1, -1, 1)$ . The metric on  $H_3$  has the form

$$ds^2 = d\bar{u}^2 - d\bar{x}^2 - d\bar{y}^2 + d\bar{v}^2, \quad (\text{A.1})$$

its isometry group is  $\text{SO}(2, 2)$ , and the scalar curvature is  $R = 6/\ell^2$ .

The space  $H_3$  can be covered by the global coordinates  $(t, \rho, \varphi)$ ,

$$\begin{aligned} \bar{u} &= \ell \cosh \rho \cos t, & \bar{x} &= \ell \sinh \rho \cos \varphi, \\ \bar{v} &= \ell \cosh \rho \sin t, & \bar{y} &= \ell \sinh \rho \sin \varphi, \end{aligned}$$

with  $t \in [-\pi, \pi]$ ,  $\rho \in [0, \infty)$ , for which the metric takes the form

$$ds^2 = \ell^2 [dt^2 \cosh^2 \rho - (d\rho^2 + \sinh^2 \rho d\varphi^2)]. \quad (\text{A.2})$$

However, since  $t$  is an angle, there are closed timelike curves in  $H_3$ . The problem can be cured by replacing the  $S^1$  time  $t \in [-\pi, \pi]$  by a new,  $R^1$  time  $t \in (-\infty, +\infty)$ , changing thereby the topology from  $S^1 \times R^2$  to  $R^3$ . The space obtained in this way is known as the *universal covering* of the AdS space. According to the commonly accepted terminology, it is this space that is called the AdS space; we denote it by  $\text{AdS}_3$ . A simple form of the  $\text{AdS}_3$  metric is obtained in the Schwarzschild-like coordinates  $r = \ell \sinh \rho$ ,  $\ell t \rightarrow t$ .

Let us now parametrize  $\text{AdS}_3$  by introducing the Poincaré coordinates:

$$\tau = \frac{-\bar{v}}{\bar{u} + \bar{x}}, \quad x = \frac{\bar{y}}{\bar{u} + \bar{x}}, \quad y = \frac{\ell}{\bar{u} + \bar{x}}.$$

They do not cover the whole space, but only one of the regions where  $\bar{u} + \bar{x}$  has a definite sign. In these regions, the metric has the form

$$ds^2 = \frac{\ell^2}{y^2} (2du dv - dy^2), \quad (\text{A.3})$$

where  $u = (\tau + x)/\sqrt{2}$ ,  $v = (\tau - x)/\sqrt{2}$ , and the boundary is located at  $y = 0$ .

The Killing vectors  $\xi = \xi^\mu \partial_\mu$  for the metric (A.3) are defined by the conditions

$$\delta_0 g_{\mu\nu} := -\partial_\mu \xi^\rho g_{\rho\nu} - \partial_\nu \xi^\rho g_{\rho\mu} - \xi^\rho \partial_\rho g_{\mu\nu} = 0.$$

They produce a set of requirements on  $\xi^\mu$ , the solutions of which define a basis of six independent AdS Killing vectors  $\xi_{(m)}$ :

$$\begin{aligned} \xi_{(1)} &= (\ell, 0, 0), & \xi_{(4)} &= (0, 2v, y), \\ \xi_{(2)} &= (0, \ell, 0), & \xi_{(5)} &= \left( \frac{u^2}{\ell}, \frac{y^2}{2\ell}, \frac{uy}{\ell} \right), \\ \xi_{(3)} &= (u, -v, 0), & \xi_{(6)} &= \left( \frac{y^2}{2\ell}, \frac{v^2}{\ell}, \frac{vy}{\ell} \right). \end{aligned} \quad (\text{A.4})$$

Turning now to the class of Siklos spacetimes (2.1), we note that it is equivalent to a subclass of Kundt spacetimes, defined by the metric

$$ds^2 = 2 \left( \frac{q}{p} \right)^2 dU (\bar{H} dU + dV) - \frac{1}{p^2} dY^2, \quad (\text{A.5})$$

where  $\bar{H} = \bar{H}(U, Y)$ , and

$$p := 1 + \frac{\lambda}{4}Y^2, \quad q := \left(1 + \sqrt{-\frac{\lambda}{4}}Y\right)^2,$$

with  $\lambda := -1/\ell^2$ . Indeed, by introducing the new coordinates

$$Y = -2\ell \frac{y + 1/2}{y - 1/2}, \quad U = 2\ell u, \quad V = 2\ell v,$$

one ends up with the Siklos metric (2.1), where the new function  $H = H(u, y)$  is defined by  $H(u, y) := \bar{H}(U, Y)|_{U=U(u), Y=Y(y)}$ .

For general  $H$ , the only Killing vector of the Siklos metric is  $\xi_{(2)} = \ell \partial_v$ , but for some specific forms of  $H$  there can be more Killing vectors; for instance,  $\xi_{(1)} = \ell \partial_u$  when  $H$  is independent of  $u$ , or the maximal number of six Killing vectors (A.4) when  $H = 0$ .

## B Improving the canonical generator

In this appendix, we construct the improved gauge generator for the massless sector of our solution.

Gauge symmetries of the first-order Lagrangian (5.1) are described by the canonical gauge generator  $G$ , the form of which can be found in eqs. (5.7) of ref. [31]. To examine the differentiability of  $G$ , we start from the form of its variation:

$$\begin{aligned} \delta G &= - \int_{\Sigma} d^2x (\delta G_1 + \delta G_2), \\ \delta G_1 &= -\varepsilon^{t\alpha\beta} \xi^\mu (b^i{}_\mu \partial_\alpha \delta \tau_{i\beta} + \omega^i{}_\mu \partial_\alpha \delta \rho_{i\beta} + \tau^i{}_\mu \partial_\alpha \delta b_{i\beta} + \rho^i{}_\mu \partial_\alpha \delta \omega_{i\beta}) + R, \\ \delta G_2 &= -\varepsilon^{t\alpha\beta} \theta^i \partial_\alpha \delta \rho_{i\beta} + R. \end{aligned} \tag{B.1}$$

Here, the variation is performed in the set of asymptotic states,  $R$  stands for regular terms and  $\rho^i$  is the Lie dual of  $\rho_{mn}$ :

$$\rho_i = 2 \left( a_0 - \frac{b_6}{6} R \right) b_i + 2b_4 \left( (Ric)_{(ik)} - \frac{1}{3} R \eta_{ik} \right) b^k.$$

Moreover, the coherently oriented volume 2-form on  $\Sigma$ , expressed in the new coordinates  $(t, \varphi, y)$ , is normalized to  $d^2x = dy d\varphi$ . Together with  $\varepsilon^{y\varphi} := \varepsilon^{ty\varphi} = 1$ , this is in accordance with the conventions used in ref. [31].

As one can see,  $G$  is not differentiable, but the problem can be corrected by going over to the improved canonical generator  $\tilde{G} := G + \Gamma$ , where the surface term  $\Gamma$  is constructed so that  $\delta \tilde{G} = R$ . In the process, transition to surface integrals is performed with the help of the Stokes formula:

$$\int_{\Sigma} d^2x \partial_\alpha v^\alpha = \int_{\partial\Sigma} df_\alpha v^\alpha = \int_0^{2\pi} d\varphi v^y, \quad df_\alpha = \varepsilon_{\alpha\beta} dx^\beta.$$

Thus, using (B.1) and the asymptotic conditions (4.5) and (4.8), the surface term  $\Gamma$  in the improved generator  $\tilde{G} \equiv G + \Gamma$  is found to have the following form:

$$\begin{aligned} \Gamma &= \Gamma_u + \Gamma_v, \\ \frac{\sqrt{2}}{\ell} \Gamma_u &= -2 \left( a_0 - \frac{b_6}{\ell^2} \right) \int_0^{2\pi} d\varphi \epsilon^u \frac{1}{y} (B^-_u - B^-_v) + 2\ell a_1 \int_0^{2\pi} d\varphi \epsilon^u \partial_y (B^-_u - B^-_v) \\ &\quad + \frac{2b_4}{\ell} \int_0^{2\pi} d\varphi \epsilon^u \left( \partial_y \Omega^-_u - \partial_u \Omega^-_y + \frac{1}{y} \frac{B^-_u}{\ell} \right), \end{aligned} \quad (\text{B.2a})$$

$$\frac{\sqrt{2}}{\ell} \Gamma_v = 2 \left( a_0 - \frac{b_6}{\ell^2} \right) \int_0^{2\pi} d\varphi \epsilon^v \frac{\ell}{y} \left( \Omega^+_u - \Omega^+_v + \frac{1}{\ell} B^+_u - \frac{1}{\ell} B^+_v \right). \quad (\text{B.2b})$$

The result for  $\Gamma_u$  is simplified with the help of the condition  $a_0 - b_6/\ell^2 - a_1 = 0$ , which is used in eq. (3.9) to define the massless sector of the torsion wave. The factors  $\sqrt{2}/\ell$  appear as an effect of the change of coordinates  $(t, \varphi) \rightarrow (u, v)$  in the components of  $B^i$  and  $\Omega^i$ .

The above construction shows that  $\tilde{G}$  is differentiable provided it is finite, and the finiteness of  $\tilde{G}$  follows from the finiteness of  $\Gamma \equiv \Gamma_u + \Gamma_v$ . The term  $\Gamma_v$  is seen to be finite directly from the adopted asymptotic conditions, whereas the finiteness of  $\Gamma_u$  depends on the validity of an additional relation:

$$- \left( a_0 - \frac{b_6}{\ell^2} - \frac{b_4}{\ell^2} \right) B^-_u + \frac{b_4}{\ell} y \partial_y \Omega^-_u = \mathcal{O}_1. \quad (\text{B.3})$$

To clarify this situation, we note that the original set of the asymptotic conditions, given in eqs. (4.5) and (4.8), can be extended using the following general principle: the expressions that vanish on-shell should have an arbitrarily fast asymptotic decrease, as no solution of the field equations is thereby lost. This principle allow us to derive the needed relation (B.3) as the  $(\mu = v, i = +)$  component of the field equation

$$\varepsilon^{\mu\nu\rho} \left( \nabla_\mu \rho_{i\nu} + \varepsilon_{ijk} b^j_\nu \tau^k_\rho \right) = 0. \quad (\text{B.4})$$

The surface terms (B.2) are used in section 5 to calculate the canonical algebra of the improved gauge generators. Note, in particular, that  $\Gamma$  vanishes on the AdS background.

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